Notes on Sovereign Debt

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Outline

- Limited enforcement and sovereign debt
  - Direct sanction models
  - Reputational models
  - Bulow-Rogoff (1989) critique
- Solution methods for DSGE models
  - Solving for nonlinear system in MATLAB
  - Dynare

Sovereign Debt

Sovereign Debt questions

Key questions about sovereign debt:

- Why/when do countries borrow?
  - Intertemporal consumption smoothing and risk-sharing.
  - Emerging markets (EMs) different from developed economies.
    - Aguiar and Gopinath (2006)
- Why/when do countries repay?
  - Incentives to repay important to know what kind of contract and risk-sharing is available.
  - Reputational concern
    - Eaton and Gersovitz (1981); Thomas and Worrall (1988, 1994)
  - Bulow and Rogoff (1989) provides rebuttal to reputational models, with caveats.
  - Propose direct sanction models as alternative.

- Are there other ways to default on debt?
  - Debt inflation and financial repression (Reinhart and Sbrancia, 2011).
  - But when is financial repression optimal? (Chari, Dovis, Kehoe WP 2016)
- What happens after a country default?
  - Renegotiation (Benjamin and Wright (2009), Cruces and Trebesch (2013))
  - How big is the cost of default? (Hebert and Schreger (2017))
- How do sovereign risks affect investment? (Aguiar et. al. (2009))
- Quantitative models
  - Aguiar and Gopinath (2006), Arellano (2008)
- Sovereign Debt Crises
  - Calvo (1988), Mendoza and Yue (2012)

Roadmap

What we will do:

- Frictionless, complete markets risk-sharing: a review.
  - Full insurance is the first best.
- Retain complete markets, but introduce limited enforcement. Check if the first best (full insurance) is still feasible.
  - Derive a deviation condition.
- If the first-best is not feasible, can we at least obtain some partial insurance?
- Assume can save post-default, how much debt can we sustain using reputational threat alone? ([**?**])

Perfect Risk-sharing Benchmark

- Home agents maximize expected utility

$$\mathbb{E}\left[u(C(\varepsilon))\right] = \int \pi(\varepsilon) u(C(\varepsilon)) d\varepsilon \tag{1}$$

  using insurance contracts:

$$C(\varepsilon) = Y(\varepsilon) - P(\varepsilon)$$

  with $Y'(\varepsilon) > 0$.

- Participation constraint for **risk-neutral** lenders:

$$\mathbb{E}\left[P(\varepsilon)\right] = \int \pi(\varepsilon) P(\varepsilon) d\varepsilon = 0$$

- Let μ be the Lagrange multiplier on participation constraint. FOC wrt $P(\varepsilon)$:

$$u'(C(\varepsilon)) = \mu \quad \forall \varepsilon$$

  i.e. $C(\varepsilon) = \mathbb{E}Y(\varepsilon)$, $P(\varepsilon) = Y(\varepsilon) - \mathbb{E}Y(\varepsilon)$.

Remarks

- Here static problem, but can easily solve dynamic problem.
- Insurance contracts can be replicated by portfolio of Arrow securities.
- Generally, full risk-sharing ensures perfect consumption growth:

$$\frac{u'(C_{t+1})}{u'(C_t)} = \frac{u'(C^*_{t+1})}{u'(C^*_t)}$$

- With risk-neutral lenders, RHS $= 1$, so perfect consumption smoothing:

$$C_{t+1} = C_t$$

Limited enforcement

- Key characteristics of sovereign debt: limited enforcement.
  - You cannot seize Santorini if Greece defaults
- Contracts must be *self-enforcing*.
  - Both parties must at all time deem it beneficial to stay within the contract.
  - No need for any third-party enforcement.
- Two types of incentives to not default:
  - To avoid direct sanction cost
  - To avoid exclusion from capital markets

Direct sanction models

- Now suppose that can seize fraction η of output in case of default:

$$\begin{aligned} \max \quad & \mathbb{E}\left[U(Y(\varepsilon) - P(\varepsilon))\right] \\ s.t. \quad & P(\varepsilon) \leq \eta Y(\varepsilon) \\ & \mathbb{E}\left[P(\varepsilon)\right] \geq 0 \end{aligned}$$

- FOC:

$$U'(C(e)) = \mu - \frac{\lambda(\varepsilon)}{\pi(\varepsilon)}$$

Complementarity slackness:

$$\lambda(\varepsilon)\left[\eta Y(\varepsilon) - P(\varepsilon)\right] = 0$$

Direct sanction models

- If $\eta Y(\varepsilon) > P(\varepsilon)^{FB} \equiv Y(\varepsilon) - \mathbb{E}Y(\varepsilon)$ for every state: first best contract is incentive compatible.
    - Would be the case if $\eta = 1$ (and $\mathbb{E}Y(\varepsilon) > 0$).
    - But if direct sanction is limited, i.e. very low η, may not get first-best.
- If FB not feasible, $\exists \varepsilon^*$ such that:
    - For $\varepsilon \leq \varepsilon^*$: $\lambda(\varepsilon) = 0$ and $U'(C(\varepsilon)) = \mu$. (IC does bot bind)
        - Consumption is equalized across states when IC does not bind.
    - For $\varepsilon > \varepsilon^*$: $\lambda(\varepsilon) > 0$, and $P(\varepsilon) = \eta Y(\varepsilon)$. $C(\varepsilon) = (1-\eta) Y(\varepsilon)$
        - Consumption increasing in region when IC binds.
        - Cannot force countries to pay too much in good states $\rightarrow$ give them more consumption to be incentive-compatible.
- Threshold ε^* pinned down from the participation constraint.

Reputation models

"If, however, mobility costs are low and enforcement costs are high a wage contract must be self-enforcing so neither the firm nor the worker ever have an incentive to renege. [...] long-run wage contracts cannot be enforced because firms and workers are unable to precommit themselves to future actions. [...] To prevent them reneging, contracts must be self-enforcing: they must offset any short-term gain from reneging by greater long-term benefits from compliance."

- Two-sided limited commitment.
- References: Thomas and Worrall (1994), Ljungqvist and Sargent textbook ch 19, 20.
- We consider only one-sided limited commitment in this course.

Is first best feasible with limited enforcement?

- Recall first-best (now dynamics)

$$U_t = \mathbb{E}_t\left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s)\right]$$

- Budget constraints

$$C_s(\varepsilon_s) + A_{s+1} = (1+r)A_s + Y(\varepsilon_s) - P_s(\varepsilon_s)$$

  Suppose no growth: $Y(\varepsilon_s) = \overline{Y} + \varepsilon_s$.

- First best: $C_s(\varepsilon_s) = \overline{Y}$, implemented by $A_s = 0$, $P_s(\varepsilon_s) = \varepsilon_s$ for all s.

Is first best feasible with limited enforcement?

- Repaying and staying gets

$$\sum_{s=t}^{\infty} \beta^{s-t} u(\overline{Y}) = \frac{u(\overline{Y})}{1-\beta}$$

- Defaulting and being in autarky from there on, gets

$$u(Y_t) + \sum_{s=t+1}^{\infty} \beta^{s-t} \mathbb{E}_t u(Y_s) = u(Y_t) + \beta \frac{\mathbb{E} u(Y_s)}{1-\beta}$$

- Will not default if and only if

$$\frac{u(\overline{Y})}{1-\beta} \geq u(Y_t) + \beta \frac{\mathbb{E} u(Y_s)}{1-\beta}$$

$$\Leftrightarrow u(Y_t) - u(\overline{Y}) \leq \frac{\beta}{1-\beta} \left[u(\overline{Y}) - \mathbb{E} u(Y + \varepsilon) \right]$$

Is first best feasible with limited enforcement?

- Full insurance is feasible when utility is not too high even in the best state

$$\lim_{Y_t \to Y_t^{max}} \left[u(Y_t) - u(\overline{Y}) \right] \leq \frac{\beta}{1-\beta} \left[u(\overline{Y}) - \mathbb{E} u(Y + \varepsilon_s) \right]$$

- Note: Y_t^{max} can well be ∞ if ε_t has unbounded support, but utility needs not be. For example,

$$\lim_{Y_t \to \infty} u(Y_t) = \lim_{Y_t \to \infty} \frac{Y_t^{1-\rho}}{1-\rho} = 0$$

if $\rho > 1$, ie. IES < 1.

Partial insurance in reputation model

- What is the optimal contract given limited enforcement?

$$\begin{aligned} \max \quad & \mathbb{E}_t\left[\sum_{s=t}^{\infty} \beta^{s-t} u(C_s)\right] \\ s.t. \quad & C_s(\varepsilon_s) = \overline{Y} + \varepsilon_s - P_s(\varepsilon_s) \\ & \mathbb{E}_{s-1} P_s(\varepsilon_s) = 0 \\ & \text{Gain}_t \leq \text{Cost}_t \end{aligned}$$

with

$$\begin{aligned} \text{Gain}_t &= u(\overline{Y} + \varepsilon_t) - u(\overline{Y} + \varepsilon_t - P(\varepsilon_t)) \\ \text{Cost}_t &= \frac{\beta}{1-\beta}\left[\mathbb{E}u(\overline{Y} + \varepsilon - P(\varepsilon)) - \mathbb{E}u(\overline{Y} + \varepsilon)\right] \end{aligned}$$

- Let $\lambda(\varepsilon_t)$ denote Lagrange multipliers on incentive constraints, μ Lagrange multipliers on participation constraints.

Partial insurance in reputation model

- Consumption is constant across all states that IC does not bind:

$$\lambda(\varepsilon_t) = 0$$

$$u'[C(\varepsilon)] = \frac{\mu}{1 + \frac{\beta}{1-\beta}\sum_{\varepsilon'}\lambda(\varepsilon')} \equiv u'(C^{nb})$$

- Consumption is higher in states that IC does bind (good states)

$$u'[C(\varepsilon)] < \frac{\mu}{1 + \frac{\beta}{1-\beta}\sum_{\varepsilon'}\lambda(\varepsilon')} = u'(C^{nb})$$

  When IC binds, get consumption from complementary slackness:

$$u(C(\varepsilon)) = u(\overline{Y}+\varepsilon) - \frac{\beta}{1-\beta}\left[\mathbb{E}u(\overline{Y}+\varepsilon' - P(\varepsilon')) - \mathbb{E}u(\overline{Y}+\varepsilon')\right]$$

- Check: $C(\varepsilon)$ is increasing in $\varepsilon \Rightarrow$ limited risk-sharing.

Summary

- With complete markets and no friction:
  - Ex-post MU growth is equalized across risk-sharing agents (equalization across states).
  - If risk-sharing with risk-neutral lenders: perfect consumption smoothing across states *and* time.
- With limited enforcement:
  - Incentive to default is highest when income is highest.
  - Full insurance generally not feasible (unless upper bound on income/utility).
  - Partial insurance is feasible and features higher consumption when income is higher.
- Note: default (on the implicit contract actually does not occur in equilibrium.

Bulow-Rogoff (1989)

Key idea:

- After defaulting, should still be allowed to write fully collateralized insurance contract.
  - including saving in a deposit and earning interest.
- If this is allowed, cannot support any positive reputational debt in equilibrium.

Bulow-Rogoff (1989): argument sketch

- There is always a day in which NPV_{payment} is higher than max debt supportable by future income.
- Default on that day.
- Every period after:
  - invest payment being demand that day into a bond,
  - post the bond and get a fully collateralized insurance contract,
  - re-invest the principal and consume the interest.
- Can show that consumption is greater than the insured consumption given by the reputational contract.

Example

- There are myriads of deviations. Here I show one very simple example:
- Suppose getting full insurance $\overline{Y}$ under the reputational contract.
  - Deviate when
    $$Y_t \geq \frac{1+r}{r}\overline{Y}$$
  - Consume $\overline{Y}$, invest $\frac{1}{r}\overline{Y}$ into a bond. (We skip the insurance contract).
  - Nex period get return $\frac{1+r}{r}\overline{Y}$, consume an extra $\overline{Y}$ (on top of the uninsured income Y_s). Re-invest $\frac{1}{r}\overline{Y}$.
  - Repeat. Consumption $> \overline{Y}$ every period (assume wlog that $Y(\varepsilon) > 0$)
- Simple deviation. Can do much better with fully collateralized insurance contract. But this shows the form of a deviation.

Other conceptual issues

So far, we have seen the reputational model of default, where:

- Incentive to default is highest when income is high.
- "Default" does not occur in equilibrium.

How do we square this with empirical facts:

- A great number of defaults occur in bad states of the world.
- We do see defaults in the real world. Is that not the equilibrium?
- Most defaults are partial, not complete.
- Countries do re-enter after some length, perhaps with renegotiation.

Grossman - Van Huyck: "Excusable default"

- Distinguish between:
  - "Excusable default": implcitly understood contingencies that countries cannot repay in bad states of the world.
  - "Inexcuseable default": unjustified repudiation.
- Excusable defaults can occur in equilibrium, but inexcusable ones are off-equilibrium.
- Compact way to put this:

$$\text{Contingent debt} = \text{Noncontingent debt} \times \text{Contingent haircuts}$$