Notes on Labor Supply and Unemployment

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Outline

- Survey of Labor Supply Elasticities
- DMP Model of Unemployment
- Shimer Puzzle and Debate on Wage Rigidity
    - Hagedorn - Manovskii (2008) calibration

Labor Supply

Labor Supply Elasticities I

- Classify elasticities by:
  - Static / "steady state" vs. dynamic
    - Static: Hicksian (substitution), Marshallian (substitution + incsome)
    - Dynamic: Frisch (intertemporal substitution)
  - Extensive vs. intensive
    - Intensive: change in aggregate hours due to currently working people
    - Extensive: change in aggregate hours due to people moving between work and non-work.

Static Labor Supply

- Static model of labor supply:

$$V(w,y) \equiv \max \quad u(c,n)$$
$$s.t. \quad c = wn + y$$

Marshallian (uncompensated) demand $c(w,y)$ and $n(w,y)$.

- Dual cost minimization problem gives Hicksian (compensated) demand $c(w,u)$ and $n(w,u)$.
- Define Marshallian and Hicksian elasticity of substitution:

$$\varepsilon^M \equiv \frac{\partial \ln n(w,y)}{\partial \ln w}, \quad \varepsilon^I \equiv \frac{\partial \ln n(w,u)}{\partial \ln w}, \quad \varepsilon^I \equiv \frac{\partial \ln n(w,y)}{\partial \ln y}$$

- The Slutsky equation implies:

$$\varepsilon^M = \varepsilon^H - \frac{s}{1-s}\varepsilon^I \qquad (1)$$

where $s \equiv wn/(wn+y)$ is the labor income share. Proof

Dynamic Labor Supply and Frisch Elasticity I

- Question: suppose relative wage between today and future change. How much is the change in labor supply today vs the future?
- Dynamic problem:

$$\max \quad \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \right]$$
$$s.t. \quad c_t + a_{t+1} = w_t n_t + (1 + r_t) a_t$$

- Perturbate the intratemporal FOC, holding marginal utility constant at $u_{c,t} = \bar{\lambda}$:

$$u_n(c_t, n_t) = -u_c(c_t, n_t) w_t = \bar{\lambda} w_t \tag{2}$$

$$du_n(c_t, n_t) = u_{nn} dn_t + u_{nc} dc_t = \bar{\lambda} dw_t$$

Dynamic Labor Supply and Frisch Elasticity II

- Assume that utility is separable between consumption and leisure, so $u_{nc} = 0$.
- The Frish elasticity is given by:

$$\varepsilon^F \equiv \frac{d \ln n_t}{d \ln w_t} = \frac{u_{n,t}}{u_{nn,t} n_t}$$

- Specific utility:

$$u(c_t, n_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} - \zeta \frac{n_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \tag{3}$$

Frisch elasticity is then constant:

$$\varepsilon^F = \frac{u_{n,t}}{u_{nn,t} n_t} = \varphi$$

  - Generally, ε^F depends on level of hours worked.
  - φ can be interpreted exactly as the intertemporal elasticity of substitution for labor.

Dynamic Labor Supply and Frisch Elasticity III

- If utility is Cobb-Douglas:

$$u(c_t, n_t) = \ln c_t + \alpha \ln(T - n_t)$$

  the Frisch elasticity is

$$\varepsilon^F = \frac{u_{n,t}}{u_{nn,t} n_t} = \frac{T - n_t}{n_t}$$

- Prescott (2004): $n_t / T \approx 0.25$, so $\varepsilon^F \approx 3$.

Substitution: Hicks vs. Frisch Elasticity

- Both the Hicksian elasticity (ε^H) and the Frisch elasticity (ε^F) measures substitution effect.
- Crucial difference: ε^H measures substitution effect in steady states, while ε^F intertemporal substitution.
  - ε^H appropriate in context of, say, cross-country difference in labor supply due to tax differences.
  - ε^F appropriate to calibrate for business cycle variation.

Labor Supply Elasticities II

Table 1—Micro vs. Macro Labor Supply Elasticities

		Intensive Margin	Extensive Margin	Aggregate Hours
Steady State	micro	0.33	0.26	0.59
(Hicksian)	macro	0.33	0.17	0.50
Intertemporal	micro	0.54	0.28	0.82
Substitution (Frisch)	macro	[0.54]	[2.30]	2.84

Note: Each cell shows a point estimate of the relevant elasticity based on meta analyses of existing micro and macro evidence. Micro estimates are identified from quasi-experimental studies; macro estimates are identified from cross-country variation in tax rates (steady state elasticities) and business cycle fluctuations (intertemporal substitution elasticities). The aggregate hours elasticity is the sum of the extensive and intensive elasticities. Macro studies do not always decompose intertemporal aggregate hours elasticities into extensive and intensive elasticities. Therefore, the estimates in brackets show the values implied by the macro aggregate hours elasticity if the intensive Frisch elasticity is chosen to match the micro estimate of 0.54. Sources are described in the appendix.

Labor Supply Elasticities III

- Micro estimates (from quasi-experimental studies) tend to be small.
- Macro estimates of Hicksian elasticities (from cross-country tax variation) are also small, agreeing with micro estimates (macro: 0.50 vs. micro: 0.59)
- In data: aggregate hours are very volatile.
  - Macro models match this moment by introducing large Frisch elasticity (2.84)
  - Stark contrast with micro Frisch elasticity (0.82).

Diamond - Mortensen - Pissarides (DMP) Model of Unemployment

DMP: Motivation

- Bulk (5/6) of aggregate hours fluctuation are due to extensive margin.
- Standard macro model hard to tractably differentiate intensive/extensive margin.
- Unemployment is important and rather volatile, so need to study it.
- Search model provides nice, tractable way to study unemployment.
    - And other labor issues: heterogeneity, on the job search, laborpolicy, etc.
    - Framework extended to other fields, e.g. money search.

DMP Model

- Workers:
  - Normalize labor force $= 1$ (ignore inactive people)
  - $u_t \in [0, 1]$ unemployed people, looking for and find jobs at rate f_t (endogenous). $1 - u_t$ employed people earn wage w_t, but potentially lose job (separation) at rate s_t (exogenous):

$$U_t = z_t + \beta \mathbb{E}_t \left[f_t W_{t+1} + (1 - f_t) U_{t+1} \right]$$
$$W_t = w_t + \beta \mathbb{E}_t \left[(1 - s_t) W_{t+1} + s_t U_{t+1} \right]$$

- Firms:
  - Unit mass of firms, posting vacancies, hiring, and producing.
  - Each employed worker (filled position) brings revenue p_t to the firm, but may separate workers at rate s_t. Each vacancy costs κ_t to post, and firms find employees at rate q_t (endogenous):

$$J_t = p_t - w_t + \beta \mathbb{E}_t \left[(1 - s_t) J_{t+1} + s_t \Xi_{t+1} \right]$$
$$\Xi_t = -\kappa_t + \beta \mathbb{E}_t \left[(1 - q_t) \Xi_{t+1} + q_t J_{t+1} \right] \tag{4}$$

  - There is free entry to posting vacancies.

Matching Function

- Matching takes place on centralized market
- Number of matches: $m_t = m(u_t, v_t)$
  - u and v are num. of unemployed people and num. of vacancies.
  - Reduced-form assumption.
  - Commonly assumed to be CRS.
- Define:
  - Market tightness: $\theta_t \equiv \frac{v_t}{u_t}$.
  - Prob. of finding a job: $f_t(\theta_t) \equiv \frac{m_t}{u_t} = m(1, \theta_t)$.
  - Prob of filling a vacancy: $q_t(\theta_t) \equiv \frac{m_t}{v_t} = m(\theta_t^{-1}, 1)$.
  - Note: $f_t(\theta_t) = q_t(\theta_t) \cdot \theta_t$. $f'(\theta) > 0$, $q'(\theta) < 0$.

Wage Determination I

- Total surplus of an employment:

$$S_t \equiv W_t - U_t + J_t - \Xi_t$$

  (in equilibrium, $\Xi_t = 0$).

- Nash bargaining picks wage that maximize:

$$\max_{w_t} \quad (W_t - U_t)^{\mu}(J_t - \Xi_t)^{1-\mu}$$
$$s.t. \quad (W_t - U_t) + (J_t - \Xi_t) = S_t$$

- μ: bargaining power of workers.
- Note that S_t is independent of wage.
- Solution:

$$\frac{W_t - U_t}{\mu} = \frac{J_t - \Xi_t}{1-\mu} = S_t$$

Wage Determination II

- Wage that supports this solution:

$$w_t = \mu(p_t + \kappa_t\theta_t) + (1 - \mu)\, z_t$$

  - $\mu = 0$: firms extracts 100% rent, and worker earns only outside option (below MPL p_t)
  - $\mu = 1$: workers extract 100% rent (MPL p_t plus the vacancy cost that the firm can avoid next period $\kappa_t\theta_t$)

DMP Equilibrium Conditions

1. Free entry to posting vacancies implies $\Xi_t = 0$ for all t. (4) implies:
$$\mathbb{E}_t J_{t+1} = \frac{\kappa_t}{\beta q_t}$$
2. Nash bargaining, which splits total surplus from an employment / filled vacancy, determines wage:
$$w_t = (1-\mu) z_t + \mu p_t + \mu \kappa_t \theta_t$$
3. Plug $w_t \to J_t$ to find job-creation condition:
$$\frac{\kappa_t}{\beta q_t(\theta_t)} = \mathbb{E}_t \left[(1-\mu)(p_{t+1} - z_{t+1}) - \mu \kappa_{t+1} \theta_{t+1} + (1 - s_{t+1}) \frac{\kappa_{t+1}}{\beta q_{t+1}} \right] \quad (5)$$
4. Finally, law of motion for u_t:
$$u_{t+1} = (1 - f_t(\theta_t)) u_t + s_t (1 - u_t) \quad (6)$$

DMP Steady State

- Job-creation condition (5) pins down market tightness in steady state:

$$s\frac{\kappa}{\beta q(\bar{\theta})} + \mu\kappa\bar{\theta} = (1-\mu)(p-z) \tag{7}$$

  In $v-u$ space, this is a straight line through the origin.

- Law of motion for u in steady state gives an inverse relationship between u and v (Beveridge curve):

$$u = \frac{s}{s+f(v/u)}$$

DMP Steady State

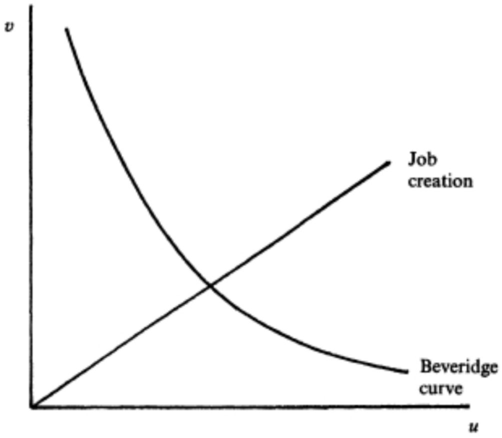

FIGURE 2
Equilibrium vacancies and unemployment

Comparative Statics: Example

- Example: what is the effect of higher worker's bargaining power (μ) on equilibrium outcomes?
- η does not shift the Beveridge curve.
- Consider the job-creation condition:

$$\Gamma(\theta, \mu; \Lambda) \equiv s\frac{\kappa}{\beta q(\theta)} + \mu\kappa\theta - (1-\mu)(p-z) = 0$$

  where $\Lambda \equiv (s, \kappa, p, z, \Phi, \alpha)$ is the vector containing remaining parameters.
- Implicit Differentiation Theorem:

$$\frac{d\theta}{d\mu} = -\frac{\Gamma_\mu}{\Gamma_\theta}$$

$$\Gamma_\mu = \kappa\theta + p - z > 0, \quad \Gamma_\theta = -\frac{s\kappa}{\beta q(\theta)^2}q'(\theta) + \mu\kappa > 0 \quad (q' < 0)$$

  Thus: $d\theta/d\mu < 0$, i.e. the job creation line becomes less steep. So: $u\uparrow$, $v\downarrow$, $\theta\downarrow$, $w\uparrow$.

Efficiency

- Is the search equilibrium efficient?
  - This is beyond inefficiencies arising from the search frictions. The question is: would the social planner do anything differently when facing the same matching function?
- When someone decides to search for jobs, it:
  - makes it easier for firms to fill a vacancy (*thick-market externality*).
  - makes it harder for another job-seeker to find a job (*congestion externality*).
- Two effects exactly cancel out when $\mu = \alpha$, and market is efficient (Hosios condition).

Can search model match data? Shimer (2005)

TABLE 1—SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1951–2003

		u	v	v/u	f	s	p
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation matrix	u	1	−0.894	−0.971	−0.949	0.709	−0.408
	v	—	1	0.975	0.897	−0.684	0.364
	v/u	—	—	1	0.948	−0.715	0.396
	f	—	—	—	1	−0.574	0.396
	s	—	—	—	—	1	−0.524
	p	—	—	—	—	—	1

TABLE 3—LABOR PRODUCTIVITY SHOCKS

		u	v	v/u	f	p
Standard deviation		0.009	0.027	0.035	0.010	0.020
		(0.001)	(0.004)	(0.005)	(0.001)	(0.003)
Quarterly autocorrelation		0.939	0.835	0.878	0.878	0.878
		(0.018)	(0.045)	(0.035)	(0.035)	(0.035)
Correlation matrix	u	1	−0.927	−0.958	−0.958	−0.958
			(0.020)	(0.012)	(0.012)	(0.012)
	v	—	1	0.996	0.996	0.995
				(0.001)	(0.001)	(0.001)
	v/u	—	—	1	1.000	0.999
					(0.000)	(0.001)
	f	—	—	—	1	0.999
						(0.001)
	p	—	—	—	—	1

Can basic search model match data?

- Shimer's Puzzle: basic search model cannot match volatility of u, v, θ in equilibrium.
  - Model generates volatility an order of magnitude lower than that observed in data.
  - In contrast, $\sigma(w)$ and $corr(w, p)$ too high in model compared to data.
- Problem in Shimer (2005): $\varepsilon_{w,p}$ too high and $\varepsilon_{\theta,p}$ too low.
  - Recall:

$$w_t = (1-\mu)z_t + \mu\left(p_t + \kappa_t\theta_t\right)$$
$$w_t - z_t = \mu(p_t - z_t + \kappa_t\theta_t)$$

  - When $p - z$ increases by 1% and suppose θ increases by 1%. Given $\mu \approx 0.72$ and $\kappa \approx 0.21$, $w - z$ goes up by almost 1%.
  - Large increase in w soaks up benefit of higher p, dampening firms' incentive to post vacancies.

Proposed solutions to Shimer (2005) Puzzle

- Hall (2005)'s solution: assume rigid wages, perhaps due to social norms.
  - But this is not an inefficient outcome, since any wage remaining in bargaining set satisfies efficiency.
- Hagedorn and Manovskii (2008)'s solution: assume $z \approx p$, so profit change at a larger rate.

Hagedorn and Manovskii (2008) I

- Hagedorn and Manovskii (2008) shows that:

$$\varepsilon_{\theta,p} = B \cdot \frac{p}{p - z}$$

  where $B \approx [1, 2]$ for reasonable calibration and any $\mu \in [0, 1]$.

- In data: $\varepsilon_{\theta,p} \approx 20$. Shimer calibration: $z \approx 0.4p$, $\varepsilon_{\theta,p} \leq 2 \cdot \frac{1}{1-0.4} \approx 3$.s
- To generate volatility: Hagedorn and Manovskii (2008) sets $z = 0.95p$.
- Intuition: $p - w$ responds little to p, so set high z so that $w \geq z$ also close to p. Smaller *level* of profits implies larger *percentage change* in profits following a productivity shocks.

Hagedorn and Manovskii (2008) II

- What about wage? Recall:

$$w_t = (1-\mu)z_t + \mu\left(p_t + \kappa_t\theta_t\right)$$
$$w_t - z_t = \mu(p_t - z_t + \kappa_t\theta_t)$$

- How to reconciles large $\varepsilon_{\theta,p} \approx 20$ with small $\varepsilon_{w,p} \approx 0.45$?
    - i.e. how to avoid the Shimer (2005) problem that wage moves by too much?
- Traditional calibration: set $\mu = \alpha$ to satisfy Hosios condition.
- Hagedorn and Manovskii (2008) instead calibrates μ to match $\varepsilon_{w,p} \approx 0.45$. Find $\mu \approx 0.052$ (much smaller than literature).

Extra Slides

Deriving equation (1)

- Slutsky equation:

$$\frac{\partial n(w,y)}{\partial w} = \underbrace{\frac{\partial n(w,u)}{\partial w}}_{\text{substitution effect}} \underbrace{-n(w,y)\frac{\partial n(w,y)}{\partial y}}_{\text{income effect}}$$

- Multiply by w/n

$$\underbrace{\frac{\partial n(w,y)}{\partial w}\frac{w}{n}}_{\varepsilon^M} = \underbrace{\frac{\partial n(w,u)}{\partial w}\frac{w}{n}}_{\varepsilon^H} - \frac{wn}{y}\underbrace{\frac{\partial n(w,y)}{\partial y}\frac{y}{n}}_{\varepsilon^I}$$

- Let $s \equiv wn/(wn+y)$. Then $wn/y = s/(1-s)$.