Notes on Global Imbalances and the Low Interest Rate Puzzle

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Related questions that need a joint answer:

- Why is the real interest rate so low in the US and other advanced economies?
- Why does capital flow from developing economies to advanced ones (south-to-north)?
- What determines the safe/risky portfolio composition of a country’s balance sheet?
Stylized facts.

Benchmark: 2-country neoclassical growth model.
  - Lucas (1990), Gourinchas and Rey (2014)
  - Metzler diagram

Heterogeneous demographics and financial frictions. Coeurdacier et al. (2015)

Asset supply explanation. Caballero et al. (2008)

Asset demand explanation. Mendoza et al. (2009)
Stylized Facts
Stylized facts

1. Real interest rates of advanced economies have been declining since mid-1980s.
2. The US has persistently run current account deficit.
   - But net foreign asset declines much less due to *valuation effect*.
3. Composition: US net holder of world risky assets, ROW net holder of safe assets.
Fact 1: declining real interest rates

Figure: Short Term Real Interest Rate, U.S., U.K., Germany and France (G4), 1920-2011

Figure from Gourinchas and Rey (2014)
Fact 2: US persistent CA deficit


Figure: US persistently runs current account deficit. But its NFA declines by less.

Figure from Gourinchas and Rey (2014).
**Figure**: US persistently runs current account deficit. But its NFA declines by less.

**Figure from Gourinchas and Rey (2014).**
Fact 3: US long position in risky assets

Figure: Advanced economies long in risky assets. Emerging markets short risky assets.

Figure from Gourinchas and Rey (2014).
Private vs. Public flows

Figure: Flows to rich economies mainly driven by public flows.

Figure from Aguiar and Amador (2009)
Benchmark: Neoclassical Growth Model
Closed Economy Neoclassical Growth Model I

Closed economy:

$$\max_{\{C_t,K_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t N_t u(C_t/N_t)$$

subject to

$$C_t + K_{t+1} = F(K_t, A_t N_t) + (1 - \delta)K_t$$

$$F(K_t, A_t N_t) = K_t^\alpha (A_t N_t)^{1-\alpha}$$

$A_t$ and $N_t$ grow at rate $g_t^A$ and $g_t^N$ respectively.
Rewrite the problem in effective units of labor:

\[
\max \sum_{t=0}^{\infty} \beta^t N_t u(\tilde{c}_t A_t)
\]

subject to

\[
\tilde{c}_t + \tilde{k}_{t+1}(1 + g_{t+1}^A)(1 + g_{t+1}^N) = \tilde{k}_{t}^\alpha + (1 - \delta)\tilde{k}_t
\]

with \(\tilde{x}_t \equiv X_t/(A_t N_t)\).
The economy governed by two equations:

\[ \tilde{c}_t + \tilde{k}_{t+1}(1 + g_A^t)(1 + g_N^{t+1}) = \tilde{k}_t^\alpha + (1 - \delta)\tilde{k}_t \quad (1) \]

\[ 1 = \beta(1 + r_{t+1}^{Aut})\frac{u'(\tilde{c}_{t+1}A_{t+1})}{u'(\tilde{c}_tA_t)} \quad (2) \]

for \( r_{t}^{Aut} \equiv f'(\tilde{k}_t) - \delta. \)

In transition: low \( \tilde{k}_t \) imply high MPK, thus high autarky rate.

Capitals should flow from capital-abundant country to capital-scarce ones.
Detour: Lucas (1990)

- Back of envelope calculation:

\[ y = Ak^\alpha \rightarrow MPK = r = \alpha A^{\frac{1}{\alpha}} y^{1-\frac{1}{\alpha}} \]

- Assuming same \( A \) and \( \alpha = 0.4 \), then:

\[ \frac{r^{India}}{r^{US}} = \left( \frac{y^{India}}{y^{US}} \right)^{1-\frac{1}{0.4}} = (15)^{1.5} = 58 \]

- Free capital mobility should imply massive capital flowing from the US to India!

- But, previously, we showed an economy in *effective units of labor*...
Detour: Lucas (1990)

Lucas does adjust for that: $(\tilde{y}$ instead of $y$)

- Human capital difference.
  - Let $h$ denotes human capital, and production is
    
    $y = Ahk^\alpha \rightarrow \tilde{y} = Ak^\alpha$.
  
  - Normalize $\tilde{y} = y/h$, so
    
    $r_{\text{India}}/r_{\text{US}} = (\tilde{y}_{\text{India}}/\tilde{y}_{\text{US}})^{-1.5} = (1/3)^{-1.5} = 5$.
  
  - Still large.

- External effect of human capital: $\tilde{y} = Ak^\alpha h^\zeta$.
  
  - Use differential in schooling, can get $r_{\text{India}}/r_{\text{US}} = 1$.
  
  - Work well across countries.
  
  - But this means zero international knowledge spillovers.

- Political risks and contracting friction.
  
  - Recall the risk of expropriation from the reputation literature.
  
  - But returns were not equalized when India was under British control before 1945.
Steady state: pair $(\tilde{c}, \tilde{k})$ constant that satisfy 1 and 2.

Assume CRRA utility.

$$\beta (1 + r^{Aut}) \frac{A_{t+1}^{-\gamma}}{A_t^{-\gamma}} = 1 \Rightarrow r^{Aut} = \frac{1}{\beta} (1 + g^A)^\gamma - 1$$

With $\beta = e^{-\rho}$, can approximate

$$r^{Aut} = \rho + \gamma g^A$$

Economies that grow faster have higher autarky interest rate.

Due to high desire to borrow from the (richer) future.

No role for demographics change here ($g^N$ not determinant of autarky rate).

Contrast with Coeurdacier et al. (2015) later.
Benchmark: Financial integration

- The budget constraint under financial integration

\[ \tilde{c}_t + (\tilde{k}_{t+1} + \tilde{b}_{t+1})(1+g_{t+1}^A)(1+g_{t+1}^N) = \tilde{k}_t^\alpha + (1-\delta)\tilde{k}_t + (1 + r_t^W)\tilde{b}_t \]

where \( \tilde{b}_t \) denotes Home NFA at the beginning of period.

- Can show via FOCs that under financial integration, returns are equalized

\[ r_t^W = r_t = r_t^* \quad \forall t \]

where \( r_t \equiv \alpha \tilde{k}_t^{\alpha-1} - \delta \), \( r_t^* = \alpha \tilde{k}_t^*^{\alpha-1} - \delta \). Thus:

\[ \tilde{k}_t = \tilde{k}_t^* \quad \forall t \]

- Exercise: how about consumption growth rate of Home compared to Foreign?
Assume \( g^A < g^*A \), so that \( r^{Aut} < r^{*,Aut} \). Can show through algebra:

\[
\begin{align*}
& r^{Aut} < r < r^{*,Aut} \\
& g^A < g^W_{cons} < g^*A \\
& \tilde{b} = -\tilde{b}^* 
\end{align*}
\]

Slow (fast) technological growth countries have low (high) autarky interest rate, export (import) capital, and accumulate foreign assets (liabilities).
Figure: Metzler diagram of financial integration
Remarks on Benchmark I

- Which countries have high autarky interest rate? Emerging or developed economies?
- In the previous analysis: developing countries grow faster, thus should have higher autarky interest rate.
- But to get capital flow South-North like the data, developing countries must have lower autarky interest rate (at least from a theory perspective).
Remarks on Benchmark II

- Developing countries can have lower autarky interest rate because:
  - Large financial friction depresses ability to borrow by young people. Coeurdacier et al. (2015)
  - Low capacity to supply asset (instruments for saving) depresses interest rate. Caballero et al. (2008)
  - Low financial friction depresses risk-sharing, increasing demand for precautionary saving, depresses interest rate. Mendoza et al. (2009)

- This class of models all point to financial development as a key factor, though the channels are somewhat different.
Models Generating Low Interest Rate in Developing Countries
Coeurdacier, Guibaud, and Jin (2015)

- Overlapping generations: young (y), middle (m), old (o).
- Utility of agent born at time $t$:
  \[
  U_t = u(c_{y,t}) + \beta u(c_{m,t+1}) + \beta^2 u(c_{o,t+2})
  \]
- One-period risk-free bond: cohort $t$ holds $a_{y,t+1}$ in period $t$ and $a_{m,t+2}$ in period $t + 1$.
- BC:
  \[
  c_{y,t} + a_{y,t+1} = w_{y,t}
  \]
  \[
  c_{m,t+1} + a_{m,t+2} = w_{m,t+1} + R_{t+1} a_{y,t+1}
  \]
  \[
  c_{o,t+2} = R_{t+2} a_{m,t+2}
  \]
  (old people have no income)
No agent can borrow more than fraction $\theta$ of NPV future income

$$a_{y,t+1} \geq -\theta \frac{w_{m,t+1}}{R_{t+1}}$$

$$a_{m,t+2} \geq 0$$

Production:

$$Y_t = A_t(e_t L_{y,t} + L_{m,t})$$

$$w_{y,t} = e_t A_t, \quad w_{m,t} = A_t$$

$e_t$: modelling device to make young poorer than middle-aged, thus incentive to borrow when young.
Optimal $a_{m,t+1}$:

$$u'(c_{m,t}) = \beta R_{t+1} u'(c_{o,t+1})$$

The Euler constraint does not bind for $m$. (Why?)

Optimal $a_{y,t+1}$:

$$u'(c_{y,t}) = \beta R_{t+1} u'(c_{m,t+1}) + \mu_{y,t}$$

$$\mu_{y,t} \left( a_{y,t+1} + \theta \frac{w_{m,t+1}}{R_{t+1}} \right) = 0$$
- Middle-aged have to save for retirement. (Asset demand)
- Young want to borrow, but are constrained. (Asset supply)
- The more severe the financial friction, the lower asset supply is. This bids up price, and depresses return.
Let $1 + g_{L,t} \equiv \frac{L_{y,t}}{L_{m,t}}$, and $1 + g_{A,t+1} \equiv \frac{A_{t+1}}{A_t}$. With $\sigma = 1$:

$$R_{t+1} = \frac{1 + \beta}{\beta} \frac{\theta}{1 - \theta} (1 + g_{L,t})(1 + g_{A,t+1})$$

Factors that lower supply relative to demand for assets: lower population growth, lower productivity growth, and more severe financial friction. (Why?)
Financial integration

- Now: two countries $U$ and $C$. They are financially integrated.
- $U$ grows slower than $C$

\[(1 + g_L^U)(1 + g_A^U) < (1 + g_L^C)(1 + g_A^C)\]  (3)

- But $C$ has terrible financial market $\theta^C < \theta^U$ that overall

\[\frac{\theta^C}{1 - \theta^C}(1 + g_L^C)(1 + g_A^C) < \frac{\theta^U}{1 - \theta^U}(1 + g_L^U)(1 + g_A^U)\]  (4)

- (3) is as in benchmark. (4) is the modification to explain the facts.
World interest rate

- Integrated world interest rate:

\[ R_{t+1}^W = (1 - \lambda_t^C)R^U + \lambda_t^C R^C \]

- World interest rate weighted average of autarky rate.
- \( C \) has a lower autarky rate.
- The world interest rate declines because \( C \) becomes more important in the world financial market:

\[ \frac{d\lambda_t^C}{dt} > 0 \]
Net foreign asset position of $C$:

$$NFA_t^C = \frac{\beta}{1 + \beta} (1 - \theta^C) A_t^C L_{m,t}^C \left(1 - \frac{R_t^C}{R_{t+1}^W}\right)$$

Since $R_{t+1}^W > R_t^C \forall t$, $NFA_t^C > 0 \forall t$.

Long-run NFA of $C$ grows at rate

$$\frac{NFA_{t+1}^C}{NFA_t^C} \xrightarrow{t \to \infty} (1 + g_A^C)(1 + g_L^C)$$
Numerical simulation:

Figure: World interest rate over time

Parameters based loosely on Coeurdacier et al. (2015): $g^U_A = 1\%$, $g^C_A = 4\%$, $g^U_L = 0$, $g^C_L = 2\%$, $\theta^U = 0.34$, $\theta^C = 0.32$, $\beta = 0.91$ (annually), $L^C_{m,0} = 6$, $L^U_{m,0} = 1$, $A^C_0 = 0.1$, $A^U_0 = 3$. 
Caballero, Farhi, Gourinchas (2008)

- Low financial development = inability to securitize available current and future outputs.
- Low asset supply depresses autarky interest rate.
- Non-Ricardian equivalence feature is important.
  - Lower ability to securitize output depresses supply of asset.
  - But this increases non-securitizable output, which increases saving (demand for asset) today if agents are Ricardian.
  - In equilibrium: level of financial development does not matter for autarky interest rate.
Continuous-time.

Continuum agents of mass 1.

Birth rate = death rate = $\theta$.

At birth, agents receive endowment $(1 - \delta)X_t$. Save entirely and consume at time of death.

- No meaningful consumption-saving decision for parsimony.

Vehicle to save: Lucas tree with dividend $\delta X_t$ and value $V_t$.

$\delta$: degree of financial development.

- $\delta = 1$: all output in the economy can be written as claims and traded.
- $\delta = 0$: no output can be written as claims.
- $\delta$ important for determining interest rate.
Financial wealth evolve according to

\[ \frac{\dot{W}_t}{W_t} = r_t - \theta + (1 - \delta) \frac{X_t}{W_t} \]

agg. wealth growth

investment return

consumed

noncapitalizable wealth

Return from saving in the Lucas tree:

\[ r_t = \delta \frac{X_t}{V_t} + \frac{\dot{V}_t}{V_t} \]

dividend rate

capital gain
In equilibrium: savings must be equaled to the value of the trees:

\[ W_t = V_t \]

This yields:

\[ W_t = \frac{X_t}{\theta} \]

\[ r_t^{Aut} = \frac{\dot{X}_t}{X_t} + \delta \theta \equiv g + \delta \theta \]

- Higher \( g \): increases high capital gain, raising interest rate.
- Higher \( \delta \): more asset supply, raising interest rate.
- Higher \( \theta \): lower financial wealth, reducing demand for asset, raising interest rate.
$$PV_t = \int_t^{\infty} e^{-\int_t^s r_{\tau} d\tau} X_s ds$$

$$V_t = \delta PV_t$$

$$N_t = (1 - \delta) PV_t$$

- Note that $r_t$ endogenous $\rightarrow$ changing $V_t$.
- Non-Ricardian equivalence is important because $\delta \uparrow \rightarrow V_t \uparrow$, but $\delta \uparrow \rightarrow N_t \downarrow$ one-for-one, which leads to $W_t \uparrow$ one for one. $r_t$ does not change.
- Customers have full rights over $V_t$, but not $N_t$. 
Small Open Economy

Define the Current Account and Trade Balance:

\[ CA_t = \dot{W}_t - \dot{V}_t \]
\[ TB_t = X_t - \theta W_t \]

\[ \frac{V_t}{X_t} = \delta \int_t^\infty e^{-r(s-t)} \frac{X_s}{X_t} ds \xrightarrow{\quad t \to \infty \quad} \frac{\delta}{r - g} \]

\[ \frac{W_t}{X_t} = W_0 e^{(r-\theta)t} + \int_0^t (1 - \delta) X_s e^{(r-\theta)(t-s)} ds \xrightarrow{\quad t \to \infty \quad} \frac{1 - \delta}{g + \theta - r} \]
Figure 2. The Metzler Diagram

\[
\frac{W}{X} = \frac{1}{g + \theta - r} \\
\text{(Demand)}
\]

\[
\frac{V}{X} = \frac{\delta}{r - g} \\
\text{(Supply)}
\]
Current Account and Trade Balance:

\[
\begin{align*}
\frac{CA_t}{X_t} & \rightarrow -g \frac{r_{aut} - r}{(g + \theta - r)(r - g)} \\
\frac{TB_t}{X_t} & \rightarrow \frac{r_{aut} - r}{g + \theta - r}
\end{align*}
\]

- If \( r_{aut} > r \), the SOE runs persistent positive trade balance, but not enough to service external liabilities.
- Thus, running persistent current account deficit.
- Got this intuition from benchmark: high autarky interest rate country imports capital and have net external liabilities.
Focus on *asset demand*.

Low financial development = endogenous incomplete market = inability to self-insure against idiosyncratic risks.

High demand for precautionary saving $\rightarrow$ low autarky interest rate.

With risky investment shock, can also explain variations in portfolio composition between countries.
Fig. 4.—Steady-state equilibria with heterogeneous financial conditions: A, autarky; B, mobility.

