Notes on Exchange Rates

Vũ T. Châu

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Questions about exchange rate

- Big debates in the exchange rate literature:
  1. If exchange rates are allowed to float, what determine it?
     - Monetary models: simple, less empirical success.
     - Recent advances: risk, rare disaster, portfolio, habit, etc.
  2. Should exchange rates be fixed or floating? Currency union?
     - Older & ongoing debate.
     - Rose (1999): The effect of currency union.
  3. Is fixed exchange rate viable in the long run?
     - In general: no if government has bad fundamentals.
     - 3 distinct models: Krugman (1979), Obstfeld (1996), Morris and Shin (1998).

Monetary Model of ER I

- "If exchange rates are allowed to float, what determine it?"
- Three ingredients: money demand, PPP, UIP.
- Money demand in Home and Foreign:

$$m_t - p_t = -\eta i_{t+1} + \phi y_t$$
$$m_t^* - p_t^* = -\eta i_{t+1}^* + \phi y_t^* \quad (1)$$

- Purchasing Power Parity:

$$p_t = p_t^* + e_t \quad (2)$$

where e_t is the price of Foreign currency in terms of Home currency.

- Uncovered Interest Parity:

$$\mathbb{E}_t(e_{t+1} - e_t) = i_{t+1} - i_{t+1}^* \quad (3)$$

Derivation

Monetary Model of ER II

- Subtract two equations in (1):

$$\eta(i_{t+1} - i^*_{t+1}) = (p_t - p^*_t) + \phi(y_t - y^*_t) - (m_t - m^*_t)$$

- Use (2) and (3), and iterate forward:

$$e_t = \frac{1}{1+\eta} \sum_{s=0}^{\infty} \left(\frac{\eta}{1+\eta} \right)^s \mathbb{E}_t \left(m_{t+s} - m^*_{t+s} - \phi \left(y_{t+s} - y^*_{t+s} \right) \right)$$

assuming no-bubble $\lim_{s \to \infty} \left(\frac{\eta}{1+\eta} \right)^s \mathbb{E}_t e_{t+s} = 0$.

Monetary Model in Continuous-Time I

- Same model in continuous time:

$$(m_t - m_t^*) - (p_t - p_t^*) = -\eta(i_{t+1} - i_{t+1}^*) + \phi(y_t - y_t^*)$$

- PPP: $p_t - p_t^* = e_t$, UIP: $i_{t+1} - i_{t+1}^* = \dot{e}_t$. (Exercise: show that $\dot{\mathcal{E}}_t / \mathcal{E}_t = \dot{e}_t$)
- For simplicity: assume $y_t = y_t^* = m_t^* = 0$. Then:

$$m_t - e_t = -\eta \dot{e}_t \tag{4}$$

which is a simple first-order nonhomogenous linear ODE. ▸ ODE Review

Monetary Model in Continuous-Time II

- No bubble solution: $c_1 = c = 0$.

$$e_t = m_t + \eta\mu \tag{5}$$

Derivation

- This is an important benchmark for the Krugman (1979) model. While the exchange rate is pegged, this hypothetical floating rate is called the "shadow exchange rate", i.e. what the exchange rate would be if the peg was abandoned on date t.

Monetary Model in Continuous-Time III

- Exchange rate is an asset price.
- Value depends on forward looking fundamentals.
- Ceteris paribus, raising Home money supply $\{m_{t+s}\}$ makes Home currency depreciate ($e_t \uparrow$). (Why?)
- Ceteris paribus, higher Foreign interest rate $\{i^*_{t+s}\}$ makes Home currency depreciate (and expected to appreciate).

Assessing Monetary Model of ER

- Limitation of monetary model of exchange rates:
  - Limited empirical success.
  - Money demand equation is behavioral assumption.
  - PPP deviation is large and persistent.
  - UIP deviation is pervasive.

Speculative Attacks Models

Speculative Attacks I

- First-generation model: Krugman (1979)
  - The peg is doomed to fail due to irresponsible fiscal policy.
    - but the paper is not about a theory of terrible public finance or a theory of whether the peg will fail.
  - The attack will occur when CB still has strictly positive foreign reserves. (Hence, the name "speculative attack")
  - Emphasis on rational, nonrandomness of attack timing.
- Second-generation model: Obstfeld (1996)
  - A peg can fail even if the government does not have terrible policies.
  - Multiple equilibria are possible.
    - Whether peg fails or not depends on agents' (rational) expectation. (Thus, the name "self-fulfilling")
- Third-generation model: Morris and Shin (1998)
  - Obtain equilibrium uniqueness by introducing strategic uncertainty.

Krugman (1979) I

- Key insight: if an attack on the peg occurs, it does so *before* the central bank runs out of reseve.
  - Money supply must be kept constant to defend the peg. Government spending must be financed by depleting foreign reserves.
  - At the time of attack: the growth rate of money discretely jumps (from 0 to the growth rate of government spending).
  - The exchange rate cannot jumps, since it implies an infinite capital gain.
  - Thus, the attack must occur when the shadow exchange rate equals the pegged value. This occurs when the CB still has a lot of foreign reserves.

Krugman (1979) II

- Money demand, UIP, PPP:

$$m_t - \varepsilon_t = -\eta \dot{\varepsilon}_t$$

- The exchange rate is currently pegged: $\varepsilon_t = \overline{\varepsilon}$, which implies $m_t = \overline{\varepsilon}$.
- Central bank's balance sheet:

$$\underbrace{B_{H,t} + \overline{\mathcal{E}} B_{F,t}}_{\text{assets}} = \underbrace{M_t}_{\text{liability}}$$

- The central bank is required to hold domestic government debt, which grows at rate μ as long as the exchange rate is fixed:

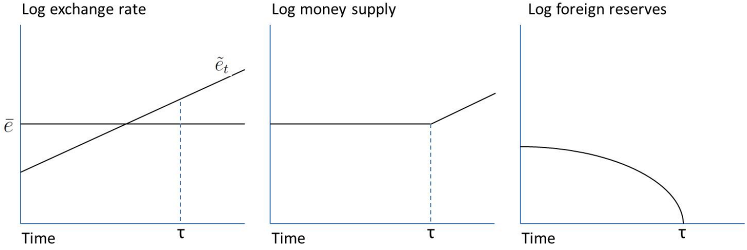
$$\frac{\dot{B}_H}{B_H} = \dot{b}_H = \mu$$

If foreign reserves run out, assume the CB grows domestic credit at rate $\frac{\dot{B}_H}{B_H} = \mu'$, and $B_F = 0$.

Krugman (1979) III

- When the exchange rate is fixed, foreign reserves is constantly depleted:
$$\dot{B}_{F,t} = -(\overline{\mathcal{E}})^{-1} \dot{B}_{H,t} < 0$$
Intuition: since the money supply is fixed to support the peg, the only way for the CB to finance domestic debt is by selling holding of foreign assets.
- Meanwhile, the shadow exchange rate (the exchange rate that would prevail when the CB runs out of foreign reserves and has to abandon the peg) is
$$\widetilde{e}_t = b_{H,t} + \eta\mu'$$
(Recall our formula (5). Why is it $b_{H,t}$ that shows up instead of m_t?)
- Caveat: we ruled out bubble solutions.

Krugman (1979): Attack occurs "before" foreign reserves run out



Figures from Krugman (1979)

- Suppose reserves are depleted by sheer force $\dot{B}_{F,t} = -(\overline{\mathcal{E}})^{-1}\dot{B}_{H,t}$.
- Let τ denote the date when reserves hit zero by this force.
- Abandoning the peg at τ implies a jump in exchange rate.
- Speculators would want to short the currency just before.
    - Borrow pesos, go to the currency board, exchange for dollars,

Obstfeld (1996): Self-fulfilling currency crisis

- Krugman (1979): speculative attack occurs because of bad government finance.
- Obstfeld (1996): self-fulfilling currency crisis can occur simply because agents expect it to fail (self-fulfilling).
- Model generates multiple equilibria:
  - Actual realization depends on a random factor that is outside of the model (sunspots). Details on sunspots
- Multiple equilibria slightly going out of fashion - but are still quite present in the literature.

Original closed-economy model: Barro and Gordon (1983)

- The model is very reduced-form, so I will not try to justify/microfound.
- The basic closed economy setting:
  - Workers have monopoly power in the labor market and can set nominal wage. Suppose the labor demand curve:

    $$y_t = \overline{y} - (w_t - p_t) - z_t$$

    where $\overline{y}$ is the natural level of output and cost-push shock z_t.
  - Efficiency: $w_t = p_t$, $y_t = \overline{y}$ (ignore z_t)
  - Wage must be set one-period in advance. Workers form rational expectation:

    $$w_t = \mathbb{E}_{t-1} p_t$$

  - This yields a Phillips Curve:

    $$\pi_t = \underbrace{\mathbb{E}_{t-1}[p_t - p_{t-1}]}_{\equiv \pi_t^e} + \underbrace{(y_t - \overline{y})}_{=x_t + k} + z_t$$

Original closed-economy model: Barro and Gordon (1983)

- Central bank's problem:

$$\max_{\pi_t, x_t} \quad x_t^2 + \chi \pi_t^2$$

$$s.t. \quad \pi_t = \pi_t^e + x_t + k + z_t$$

- Can show that if the CB can commit, it would commit to $\pi_t^* = \frac{z_t}{1+\chi}$.
- However, with output bias, it chooses $\pi_t = \frac{k}{\chi} + \frac{z_t}{1+\chi} > \pi_t^*$.
  - Government has dynamic inconsistentcy problem.
  - Want to inflate to generate output ex-post.
  - But worker has rational expectation and "prices the bias" into nominal wage: $\pi_t^e = k/\chi$.
  - In equilibrium, the inability to commit leaves the government worse-off.

Small Open Economy: Obstfeld (1996)

- For an SOE: PPP implies $\pi_t = e_t - e_{t-1}$, assuming constant foreign price.
- Central bank's problem:

$$\max_{\pi_t, x_t} \quad x_t^2 + \chi \pi_t^2$$
$$s.t. \quad \pi_t = \pi_t^e + x_t + k + z_t$$

- Same as before: government has temptation to inflate ex-post.
- With PPP, the only way to generate inflation is to let go of the peg.
- Question: can the CB credibly commit to pegging, i.e. $\pi_t = 0$?

- Taking π^e as given, optimal ex-post inflation is

$$\pi_t = \frac{\pi_t^e + k + z_t}{1+\chi}$$
$$x_t = -\frac{\chi(\pi_t^e + k + z_t)}{1+\chi}$$

which delivers minimized loss

$$\mathcal{L}^{flex} = x_t^2 + \chi\pi_t^2 = \frac{\chi}{1+\chi}(\pi_t^e + k + z_t)^2$$

- Keeping the peg implies $\pi_t = 0$, $x_t = -(\pi_t^e + k + z_t)$, thus loss:

$$\mathcal{L}^{fixed} = (\pi_t^e + k + z_t)^2 > \mathcal{L}^{flex}$$

- The government can NOT credibly commit to pegging.

With fixed cost of abandoning peg

- Now add fixed cost to the model:

$$\max_{\pi_t, x_t} \quad x_t^2 + \chi\pi_t^2 + C(\pi_t)$$
$$s.t. \quad \pi_t = \pi_t^e + x_t + k + z_t$$

where $C(\pi_t) = C(\pi_t) = \begin{cases} \overline{c}, & \pi_t > 0 \\ \underline{c}, & \pi_t < 0 \end{cases}$.

- The fixed cost should prevent the CB from inflating if z_t not too large.
- Expect:
  - If π^e is low, then real wage is low (under peg), output is relatively high, and CB has less temptation to inflate (given the fixed cost), unless z_t is very large. Average inflation is then low, confirming low π^e.
  - The opposite case if π^e is high.

Multiple equilibria

In equilibrium with rational expectation: $\mathbb{E}\pi = \pi^e$.

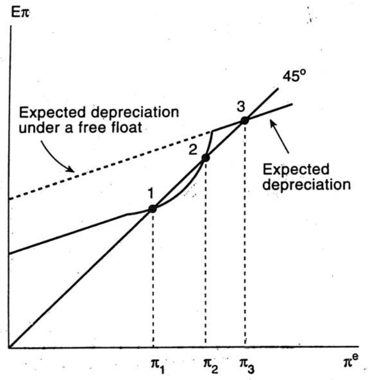


Figure: From Obstfeld (1996)

Necessity of ingredients

- Discrete adjustment cost generates sufficiently strong strategic complementarity:
  - Create a set in which the government will not inflate *as long as* expected inflation is not too high.
  - Otherwise government inflates all the time, as we've seen.
- Output bias needs to be sufficiently high to ensure equilibrium (3) (and multiplicity) exists.
  - Intuitively: the government's dynamic inconsistency problem must be sufficiently bad that if the market expects the government to float, thus π^e is high, and output is low, and the government has enough output bias that they would actually abandon the peg.

Equilibrium Uniqueness using Global Games:Morris and Shin (1998)

- Previous philosophy: speculative attacks are self-fulfilling.
- Multiple equilibria are:
  - Quite uncomfortable.
  - Why do attacks occur when they occur?
- Adding a very small noise to observations of fundamentals by agents yield equilibrium uniqueness.
- What matters is not only what one agent thinks she has accurate observation or not, but also what she thinks others think, what she thinks that others think that she thinks, etc.

Morris and Shin (1998) Setup

- Fundamental $\theta \in [0, 1]$.
- Private information: $x_i = \theta + \varepsilon_i$, with $\varepsilon_i \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, $\lambda_\varepsilon = 1/\sigma_\varepsilon^2$.
- Trader gets 0 if does not attack. If attack:

$$\text{payoff} = \begin{cases} -c & , \text{ if } R = 0 \\ 1 - c & , \text{ if } R = 1 \end{cases}$$

where $R = 1$ iif the peg is abandoned, and $c \in (0, 1)$.

- Flat prior: $\mathbb{P}(\theta = \theta') = 1$ for all $\theta' \in [0, 1]$.
- The government abandons the peg if the size of attack A is sufficiently large. In particular:

$$R = 1 \Leftrightarrow A \equiv \int_0^1 I_j dj > \theta$$

where I_j is the action of trader j.

Perfect information benchmark

- Suppose $\sigma_\varepsilon^2 = 0$, so every knows θ perfectly.
- Multiple equilibria for $\theta = [0, 1)$:
  - $I_j = 1 \, \forall j$, $A = 1$, $R = 1$.
  - $I_j = 0 \, \forall j$, $A = 0$, $R = 0$.
- Strategic complementarity is key:
  - If you attack, I will attack, and the government abandons. Vice versa.
  - Perfect information and common knowledge is necessary, since coordination between all agents (including the CB) is required.
  - Adding noise weakens strategic complementarity. (When I attack, I no longer know for sure that you will attack, and so on.)

Imperfect information

- Go back to the imperfect information case.
- Solution strategy:
  - Step 1: Guess a cutoff x^* for the cutoff strategy: $I_i = 1_{x \leq x^*}$. Solve for the size of attack given x^*: $A(\theta; x^*)$.
  - Step 2: Solving for a threshold of fundamental θ^* below which the attack succeeds:

$$A(\theta^*(x^*); x^*) = \theta^* \tag{6}$$

  - Step 3: Solve for the x^* that indeed make it the cutoff for trader's strategy. In other words, the marginal trader that gets $x_i = x^*$ must be indifferent between attacking or not:

$$\mathbb{E}_{\theta^*} u(x^*) = 0 \tag{7}$$

- We will see that (x^*, θ^*) is unique.

Step 1

- The probability of getting a private signal that is below the cutoff is:
$$\mathbb{P}\left(x_i < x^*|\theta\right) = \Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta)\right)$$
- By the Law of Large Number, this is also the fraction of agents that get $x_i < x^*$ and chooses to attack. Thus the size of attack is also

$$A(\theta; x^*) = \Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta)\right)$$

Step 2

- Since $A_\theta(\theta; x^*) < 0$, $A(0; x^*) < 1$, $\exists! \theta^* : A(\theta^*; x^*) = \theta^*$. In particular, θ^* is defined implicitly as:

$$x^* = \theta^*(x^*) + \frac{\Phi^{-1}(\theta^*(x^*))}{\sqrt{\lambda_\varepsilon}} \tag{8}$$

- (Hint: try draw a diagram to see uniqueness of θ^*)

Step 3

- Given θ^*, the expected payoff for a trader with signal x_i is

$$\begin{aligned}\mathbb{E}\left[u|x_i,\theta^*\right] &= -c + 1\cdot\left(A(\theta) > \theta|x_i,\theta^*\right)\\ &= -c + 1\cdot\mathbb{P}\left(\theta < \theta^*|x_i,\theta^*\right)\end{aligned}$$

- Given flat prior, we have $\theta|x_i \sim \mathcal{N}(x_i, \lambda_\varepsilon^{-1})$. Thus:

$$\begin{aligned}\mathbb{E}[u|x_i,\theta^*] &= -c + \Phi\left(\sqrt{\lambda_\varepsilon}(\theta^* - x_i)\right)\\ &= -c + 1 - \Phi\left(\sqrt{\lambda_\varepsilon}(x_i - \theta^*)\right)\end{aligned}$$

- The marginal trader ($x_i = x^*$) must be indifferent between attacking and not:

$$
\begin{aligned}
\mathbb{E}\left(u | x^*, \theta^*\right) &= 0 \\
\Leftrightarrow \Phi\left(\sqrt{\lambda_\varepsilon}(x^* - \theta^*)\right) &= 1 - c \\
\Leftrightarrow x^* &= \frac{\Phi^{-1}(1-c)}{\sqrt{\lambda_\varepsilon}} + \theta^*
\end{aligned} \tag{9}
$$

Equilibrium

- In equilibrium, (x^*, θ^*) are characterized by (8) and (9). This implies:

$$\theta^* = 1 - c$$
$$x^* = \frac{\Phi^{-1}(1-c)}{\sqrt{\lambda_\varepsilon}} + 1 - c$$

- This equilibrium is unique.

Discussion

- Uniqueness of equilibrium retains even as $\sigma_\varepsilon^2 \to 0, \lambda_\varepsilon \to \infty$ (very close to perfect information benchmark).
- Yet, at $\sigma_\varepsilon^2 = 0$, there are multiple equilibria $\to$ discontinuity in model's properties.
- Adding a tiny amount of noise creates strategic uncertainty, and this is sufficient to break multiple equilibria.

Discussion

- Uniqueness allows analysis of equilibrium (comparative statics, simulation etc.).
- With multiple equilibria: have to select which equilibrium.

Can we rid multiple equilibria for good?*

- Angeletos et al. (2006): add a signaling game to global game.
  - Policy choice often conveys information.
  - Exogenous information asymmetry helps select unique equilibrium. Endogenous information leads to multiple equilibria.
  - Suppose we add one stage before the global game studied. The central bank can take actions to obtain better fundamentals (borrowing extra reserves, for example).
  - Multiple equilibria:
    - Inactive policy equilibrium: agents coordinate on a strategy that is insensitive to the policy. CB chooses cost minimizing policy regardless of types.
    - Active policy equilibria (continuum of them): agents play less aggressively when policy is high. CB then optimally raises policy only for intermediate types (for same logic as the tripartite in Morris and Shin (1998)). But the government's action signals to agents that she is of intermediate types.
    - Multiple equilibria are back...

Extra Slides

Effects of Currency Union: Rose (1999)

- Currency union clearly has costs, among them loss of monetary policy independence. But what are the benefits?
- Add exchange rate volatility and currency union membership to a gravity model of trade.
- Find that membership in a currency union is associated with an increase in bilateral trade by 300%!
- Robust positive, significant effect; albeit smaller in some specifications.
- Caveats:
  - Exclude observations with zero bilateral trade (which is regularity in the data) $\longrightarrow$ selection bias.
  - Simultaneity: membership in a currency union may well be endogenous to the existence of a pre-existing trade relationship.

Detour: ODE Review

Lemma

First-order nonhomogenous linear ODE of the form

$$\dot{x}(t) + A(t)x(t) = B(t) \tag{10}$$

has the general solution

$$x(t) = \frac{1}{U(t)}\left(\int U(t)B(t)dt + c\right) \tag{11}$$

where $U(t) \equiv \texttt{exp}\left(\int A(t)dt\right)$ *is the integrating factor, and* c *a constant.*

Proof.

Multiply both sides of 10 by $U(t)$. Note that LHS is:

$$U(t)\dot{x}(t) + \underbrace{U(t)A(t)}_{=\dot{U}(t)} x(t) = \frac{d}{dt}[U(t)x(t)]$$

Integrate both sides:

$$U(t)x(t) = \int U(t)B(t)dt + c$$

Divide $U(t)$ over to the other side gives us (11).
In general, some boundary condition pins down c. □

Float Exchange Rate in Continuous-time (cont.)

- Rewrite (4):

$$\dot{e}_t - \frac{1}{\eta} e_t = -\frac{1}{\eta} m_t$$

- Using 11, the general solution to 4 is

$$\begin{aligned} e_t &= \exp\left(\frac{1}{\eta} t\right) \left[\int \exp\left(-\frac{1}{\eta} t\right)\left(-\frac{1}{\eta} m_t\right) dt + c\right] \\ &= \exp\left(\frac{1}{\eta} t\right) \left[\exp\left(-\frac{1}{\eta} t\right) m_t - \int \exp\left(-\frac{1}{\eta} t\right) \dot{m}_t dt + c\right] \end{aligned}$$

- Suppose money grows at a fixed rate $\dot{m}_t = \mu$, then:

$$\begin{aligned} e_t &= \exp\left(\frac{1}{\eta} t\right) \left[\exp\left(-\frac{1}{\eta} t\right) m_t - \mu \int \exp\left(-\frac{1}{\eta} t\right) dt + c\right] \\ &= \exp\left(\frac{1}{\eta} t\right) \left[\exp\left(-\frac{1}{\eta} t\right) m_t + \eta\mu \exp\left(-\frac{1}{\eta} t\right) + c_1 + c\right] \\ &= m_t + \eta\mu + (c_1 + c) \exp\left(\frac{1}{\eta} t\right) \end{aligned}$$

Deriving UIP condition

- Write down any utility maximizing model in which an agent can freely between Home and Foreign bonds:

$$1 = \beta(1 + i_{t+1})\mathbb{E}_t\left[M_{t+1}\right]$$
$$1 = \beta(1 + i^*_{t+1})\mathbb{E}_t\left[M_{t+1}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right]$$

which implies

$$\mathbb{E}_t\left[M_{t+1}\left((1 + i_{t+1}) - (1 + i^*_{t+1})\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)\right] = 0$$

- If ignoring $\text{cov}_t\left(M_{t+1}, (1 + i_{t+1}) - (1 + i^*_{t+1})\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right)$ (for example, linearizing):

$$1 + i_{t+1} = (1 + i^*_{t+1})\mathbb{E}_t\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}$$

- In logs: $i_{t+1} - i^*_{t+1} = \mathbb{E}_t(e_{t+1} - e_t)$

Detour: sunspots?

- William Stanley Jevons publishes a paper in *Nature* in 1878 studies the correlation between business cycles and sunspots!
  - Literally the number of spots on the literal sun...

NATURE [*Nov.* 14, 1878

ars ago. er to the a con- n of the period in od. Mr. work or y, Com- s of the depres- steemed ked the disturb- , which neration nportant

money market, though often apparently due to exceptional and accidental events, such as wars, great commercial failures, unfounded panics, and so forth, yet do exhibit a remarkable tendency to recur at intervals approximating to ten or eleven years. Thus the principal commmercial crises have happened in the years 1825, 1836-9, 1847, 1857, 1866, and I was almost adding 1879, so convinced do I feel that there will, within the next few years, be another great crisis. Now if there should be in or about the year 1879, a great collapse comparable with those of the years mentioned, there will have been five such occurrences in fifty-four years, giving almost exactly eleven years (10·8) as the average interval, which sufficiently approximates to 11·1, the supposed exact length of the sun-spot period, to warrant speculations as to their possible connection."

Detour: sunspots?

- More generally, sunspots mean the economy depends on an exogenous variables that is not a part of any relationship at all:

$$\mathbb{E}_t(\mathbf{s_t}, \mathbf{x_t}, \mathbf{s_{t+1}}, \mathbf{x_{t+1}}; \zeta_\mathbf{t}) = 0$$

  where $(\mathbf{s_t}, \mathbf{x_t})$ are endogenous and exogenous state variables; ζ_t is sunspot.
- An equilibrium occurs only because people expect that it will occur.
  - everyone looks at the sun, counts the number of spots, and know what actions to play.

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